Math 5 – Trigonometry – fall '07 – Chapter 3 Test Name____________________

Show all work for credit. Write all responses on separate paper.

1. Consider the line passing the points (20,100) and (7,9) in the *x*-*y* Cartesian coordinate plane.
   a. Show that an equation for the line in *x* and *y*.
   b. Find an equation for the line parallel to this line and passing through (0,8).
   c. Find an equation for the line perpendicular to this line and passing through (0,0).

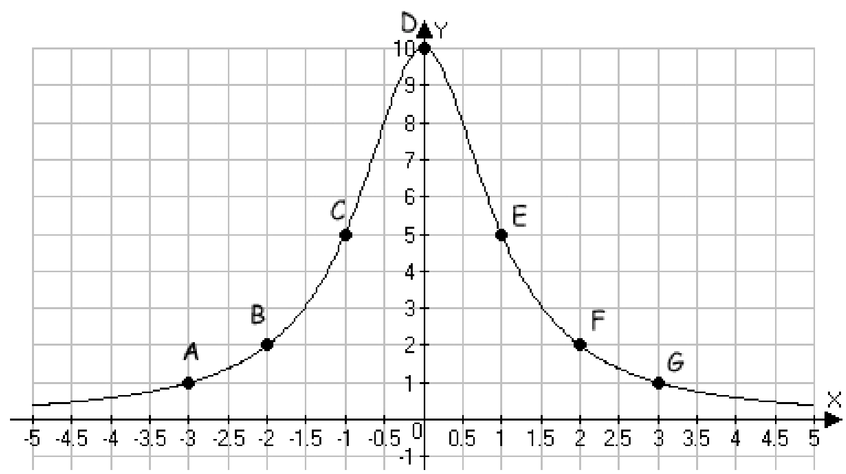
2. Consider the quadratic $f(x) = -x^2 + 2x + 2$
   a. Express the quadratic function in standard form.
   b. Express the zeros (*x*-intercepts) of the parabola in simplest radical form.
   c. Sketch its graph, showing the coordinates of the vertex and all intercepts.

3. Consider the circle with diameter extending from (0,0) to (16,30).
   a. Find the center of the circle.
   b. Find the radius of the circle.
   c. Write an equation for the circle.

4. Suppose $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x^2 - 4}$. Find the domain of $(g \circ f)(x)$

5. Compute and simplify the average rate of change of $f(x) = \dfrac{x}{x^2+1}$ over the given interval. Simplify your result.
   a. [0, 2]
   b. [1, 1+*h*]

6. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of points at *A*, *B*, *C*, *D*, *E*, *F* and *G*, and plot the given transformation
   a. $y = 2f(x)$
   b. $y = 1 + f(x-2)$
   c. $y = 10 - f(x)$



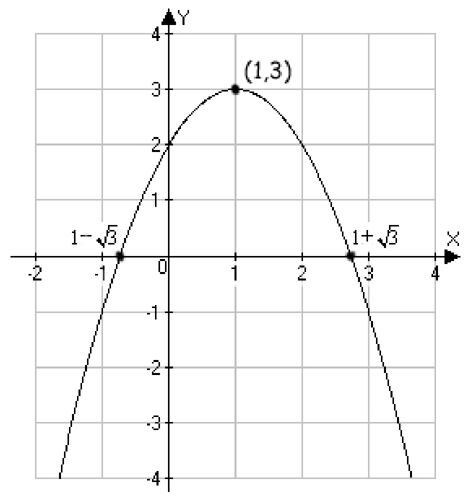
7. The total surface area of a cylinder is π square units.
   a. Find a function that models the cylinder's height as a function of its radius.
   b. Find a function that models the cylinder's radius as a function of its height.

8. Find a formula for the inverse function of $f(x) = \sqrt[3]{x+8}$ and plot the function and its inverse together in the same coordinate plane, showing the symmetry of these function across the line $y = x$.

Math 5 – Trigonometry – fall '07 – Chapter 3 Test   Solutions

1. Consider the line passing the points (20,100) and (7,9) in the x-y Cartesian coordinate plane.
   a. Show that an equation for the line in x and y.|
      ANS: The slope is $m = \frac{100-9}{20-7} = \frac{91}{13} = 7$. Plugging into the point-slope formula we have $y-9 = 7(x-7) \Leftrightarrow y = 7x - 40$
   b. Find an equation for the line parallel to this line and passing through (0,8). ANS: $y = 7x + 8$
   c. Find an equation for the line perpendicular to this line and passing through (0,0).
      ANS: $y = \frac{-1}{7}x$

2. Consider the quadratic $f(x) = -x^2 + 2x + 2$
   a. Express the quadratic function in standard form.
      ANS: $f(x) = -(x-1)^2 + 3$
   b. Express the zeros (x-intercepts) of the parabola in simplest radical form.
      ANS: $(x-1)^2 = 3 \Leftrightarrow x - 1 = \pm\sqrt{3} \Leftrightarrow x = 1 \pm \sqrt{3}$
   c. Sketch its graph, showing the coordinates of the vertex and all intercepts. ANS: (at right)



3. Consider the circle with diameter extending from (0,0) to (16,30).
   a. Find the center of the circle.
      ANS: The midpoint is (8,15).
   b. Find the radius of the circle. ANS: The radius is $\sqrt{8^2+15^2} = \sqrt{289} = 17$
   c. Write an equation for the circle. ANS: $(x-8)^2 + (y-15)^2 = 289$

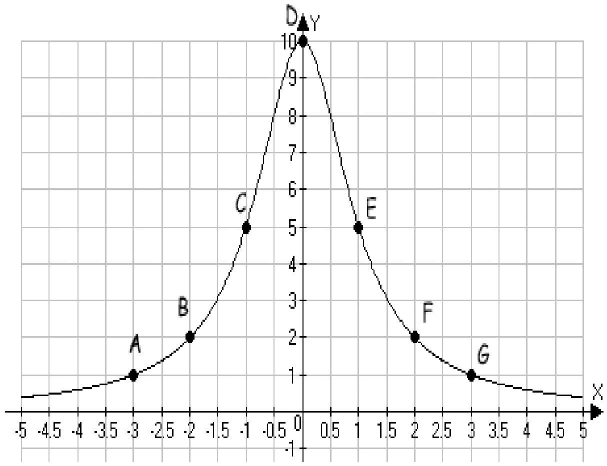
4. Suppose $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x^2-4}$. Find the domain of $(g \circ f)(x)$
   ANS: $(g \circ f)(x) = g\left(\sqrt{x+1}\right) = \frac{1}{\sqrt{x+1}^2 - 4} = \frac{1}{x-3}$ has domain $[-1,3) \cup (3,\infty)$.

5. Compute and simplify the average rate of change of $f(x) = \frac{x}{x^2+1}$ over the given interval.
   a. [0, 2] ANS: $\frac{f(2)-f(0)}{2-0} = \frac{\frac{2}{2^2+1} - \frac{0}{0^2+1}}{2-0} = \frac{1}{5}$
   b. [1, 1+h] ANS:

$$\frac{f(1+h)-f(1)}{1+h-1} = \frac{\frac{1+h}{(1+h)^2+1} - \frac{1}{1^2+1}}{h} = \frac{1+h}{h\left((1+h)^2+1\right)} - \frac{1}{2h} = \frac{2+2h-(1+h)^2-1}{2h\left((1+h)^2+1\right)}$$

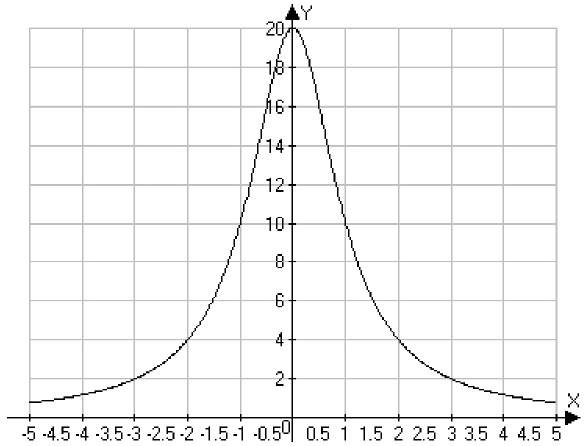
$$= \frac{2+2h-1-2h-h^2-1}{2h\left((1+h)^2+1\right)} = \frac{-h^2}{2h\left((1+h)^2+1\right)} = \frac{-h}{2\left((1+h)^2+1\right)}$$

6. Given the graph of $y = f(x)$ shown at right and the given transformation, tabulate the transformed coordinate values of points at A, B, C, D, E, F and G, and plot the given transformation.



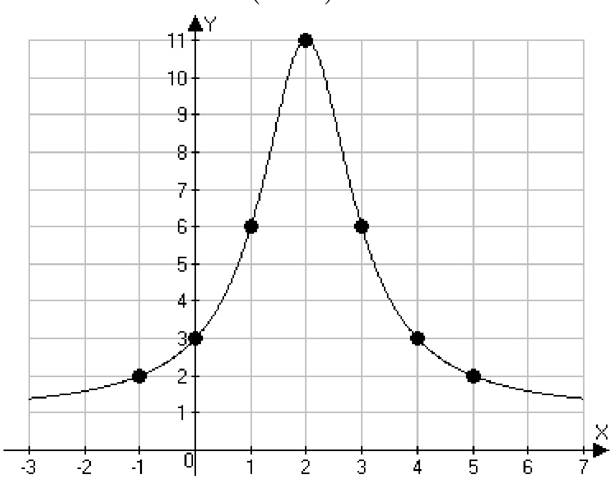
	A	B	C	D	E	F	G
x	-3	-2	-1	0	1	2	3
y	1	2	5	10	5	2	1

a. $y = 2f(x)$



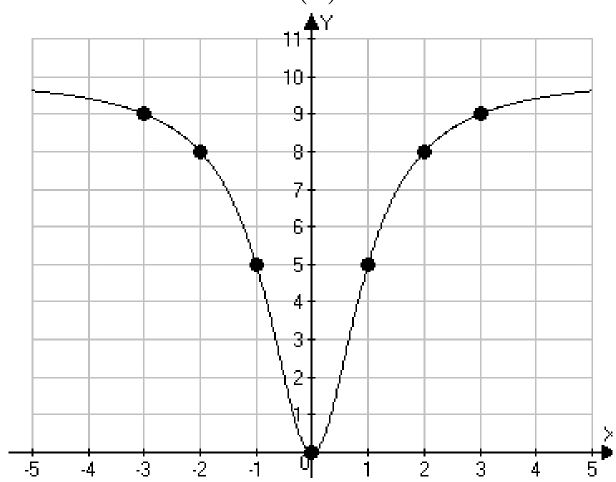
	A	B	C	D	E	F	G
x	-3	-2	-1	0	1	2	3
y	2	4	10	20	10	4	2

b. $y = 1 + f(x-2)$



	A	B	C	D	E	F	G
x	-1	0	1	2	3	4	5
y	2	3	6	11	6	3	2

c. $y = 10 - f(x)$



	A	B	C	D	E	F	G
x	-3	-2	-1	0	1	2	3
y	9	8	5	0	5	8	9

7. The total surface area of a cylinder is π square units.
   a. Find a function that models the cylinder's height as a function of its radius.

   ANS: $2\pi r^2 + 2\pi rh = \pi \Leftrightarrow 2r^2 + 2hr = 1 \Leftrightarrow 2hr = 1 - 2r^2 \Leftrightarrow \boxed{h = f(r) = \frac{1-2r^2}{2r}}$

b. Find a function that models the cylinder's radius as a function of its height.

ANS:

$$2\pi r^2 + 2\pi rh = \pi \Leftrightarrow 2r^2 + 2hr = 1 \Leftrightarrow r^2 + hr = \frac{1}{2} \Leftrightarrow r^2 + hr + \frac{h^2}{4} = \frac{1}{2} + \frac{h^2}{4}$$

$$\Leftrightarrow \left(r + \frac{h}{2}\right)^2 = \frac{h^2+2}{4} \Leftrightarrow r + \frac{h}{2} = \pm\sqrt{\frac{h^2+2}{4}} \Leftarrow \boxed{r = g(h) = \frac{h + \sqrt{h^2+2}}{2}}$$

8. Find a formula for the inverse function of $f(x) = \sqrt[3]{x+8}$. Plot the function and its inverse together in the same coordinate plane, showing the symmetry of these function across the line $y = x$.

ANS: The inverse function is $f^{-1}(x) = x^3 - 8$. We can make a table of integer points for f:

x	-9	-8	-7	0
$f(x)$	-1	0	1	8

and then simply reversing these gives the table for the invers function:

x	-1	0	1	8
$f^{-1}(x)$	-9	-8	-7	0

Plotting the points and keeping in mind the basic sigmoid (ESS) shape of the curves, we'd plot something like the following: